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VASAVI COLLEGE OF ENGINEERING (Autonomous), HYDERABAD BE I Year II - Semester (Main) Examinations, July - 2015

Mathematics - II
Time: $\mathbf{3}$ hours
Max. Marks: 70
Note: Answer ALL questions in Part-A and any FIVE questions from Part-B

## Part-A (10 X 2=20 Marks)

1. Find the unit normal to the surface $x y^{3} z^{3}=4$ at $(-1,-1,2)$
2. Explain concept of line integral.
3. Find the curl of vector function $\bar{F}=(x-y) \mathbf{i}+(y-z) \mathbf{j}+(z-x) \mathbf{k}$.
4. Define orthogonal trajectories of the family of curves.
5.* Solve the differential equation $y^{\prime \prime \prime}+y=0$.
5. Find particular integral of $y^{\prime \prime}-2 y^{\prime}+4 y=e^{x} \cos x$.
6. State Gauss Divergence theorem.
7. Evaluate $\frac{1}{\sqrt{2 \pi}} \int_{-\infty}^{\infty} e^{-x^{2} / 2} \mathrm{dx}$ by using gamma function.
8. Express $J_{5}(\mathrm{x})$ in terms of $J_{0}(x)$ and $J_{1}(x)$
9. Express Legendre's polynomials $4 \mathrm{P}_{3}(x)+6 \mathrm{P}_{2}(x)-3 \mathrm{P}_{1}(x)-2 \mathrm{P}_{0}(x)$ in powers of x .

## Part-B (5 X 10=50 Marks)

11. a) Find the Directional derivative of the scalar function $\phi=2 x^{2}+y^{2}+z^{2}$ at $(1,2,3)$ in the direction of the line $\frac{x}{3}=\frac{y}{4}=\frac{z}{5}$
b) Evaluate $\iint_{S} \bar{A} \cdot \bar{n} d s$ where $\bar{A}=\mathrm{zi}+\mathrm{xj}-3 \mathrm{y}^{2} \mathrm{z} \mathbf{k}$ and S in the surface of the cylinder $x^{2}+y^{2}=16$ included in the first octant between $z=0$ and $z=5$.
12. a) Find the orthogonal trajectories of the family of confocal conics $\frac{x^{2}}{a^{2}}+\frac{y^{2}}{a^{2}+\lambda}=1$ where $\lambda$ is the parameter.
b) Find the general solution of the differential equation $y^{\prime}=y^{2}-(2 x-1) y+x^{2}-x+1$, if $y=1$ is a solution of the equation.
13. a) Solve $\frac{d^{2} y}{d x^{2}}-4 y=x \sinh x$.
b) Show that the frequency of free vibration in a closed electrical circuit with inductance [5] L and capacity C in series is $\frac{30}{\pi \sqrt{L C}}$ per minute.

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14. a) Find the series solution of the differential equation $y^{\prime \prime}+x y=0$ about $x_{0}=1$.
b) Show that $P_{n}^{\prime}(x)=x P_{n-1}^{\prime}(\mathrm{x})+n P_{n-1}(\mathrm{x})$
15. a) Show that $J_{n}(x)=\frac{1}{\pi} \int_{0}^{\pi} \cos (n \theta-x \sin \theta) d \theta, n$ being an integer.
b) Discuss orthogonality of Bessel function.
16. a) Prove $\nabla^{2} f(r)=f^{\prime \prime}(r)+\frac{2}{r} f^{\prime}(r)$.
ab) Show that the family of curves $y^{2}=4 \mathrm{c}(x+c)$ is a self orthogonal. where c is a parameter.
17. Answer any two of the following:
a) Solve the differential equation $y^{\prime \prime}+y^{\prime}-2 y=x^{2} \sin x$
b) Express $f(x)=x^{4}+3 x^{3}-x^{2}+5 x-2$ in terms of Legendre's polynomials.
c) Prove that $J_{\frac{5}{2}}(x)=\sqrt{\frac{2}{\pi x}}\left\{\frac{3-x^{2}}{x^{2}} \sin x-\frac{3}{x} \cos x\right\}$
